Statistical Properties of Exchange Rates

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Introduction

- Future fluctuations in exchange rates may result in potential loss and possible gain
- When delivery and payment for goods or services do not coincide there may be some form of transaction exchange risk
- Future spot exchange rate cannot be known with certainty
- Need to quantify possible future changes in the exchange rate
- Allows us to measure the degree of risk associated with international financial transactions

Assessing Exchange Rate Uncertainty

- Analysis of historical data can be used to derive important insights
- Serves as an important gauge for what may happen in the future
- Consider histogram of monthly % changes in GBPUSD and associated normal distribution
- To compute monthly % changes in the exchange rate

$$s_t = \frac{(S_t - S_{t-1})}{S_{t-1}}$$

Assessing Exchange Rate Uncertainty

Distribution of Daily Changes in GBPUSD

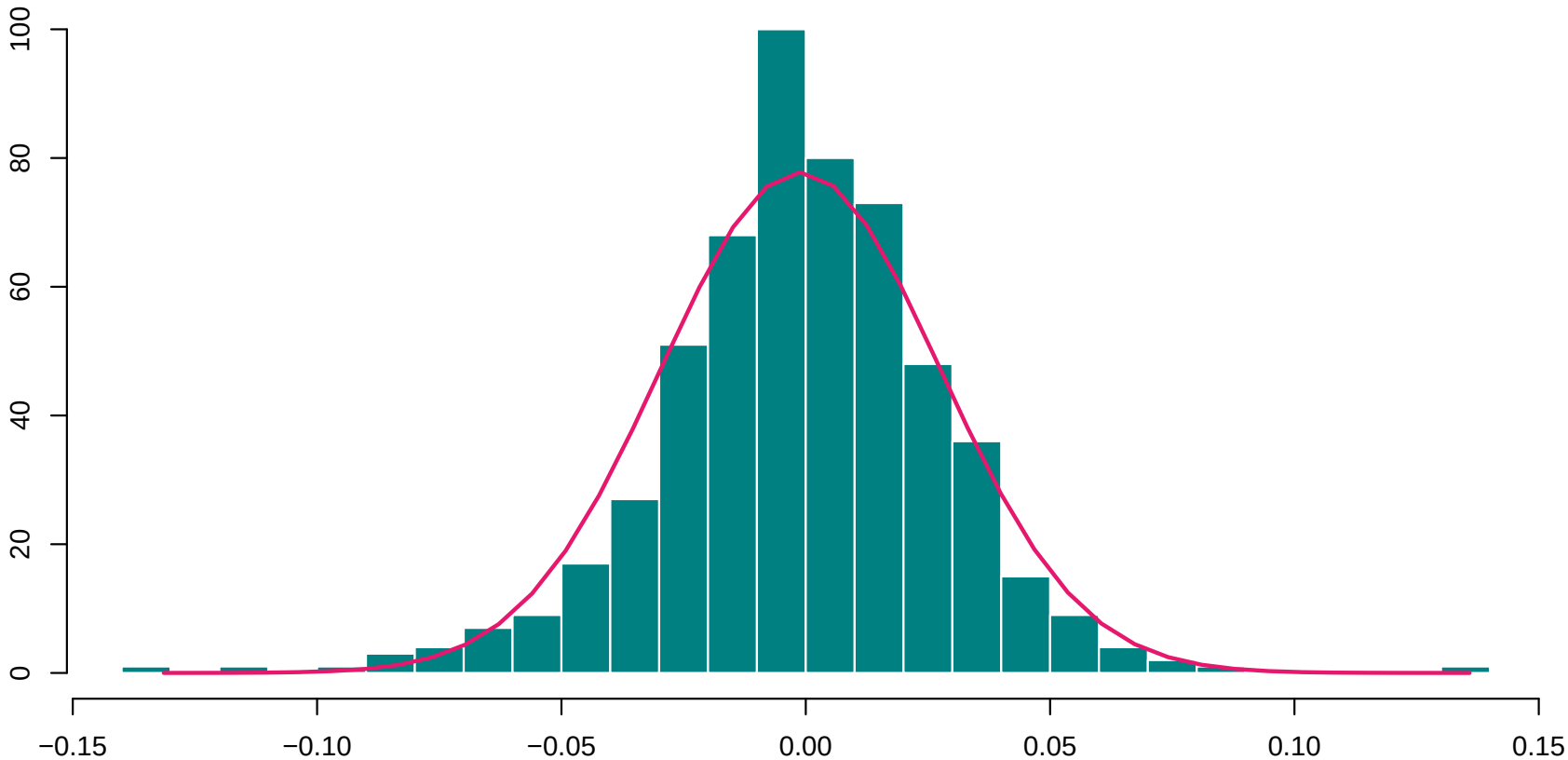

Figure: Source: Bloomberg

Assessing Exchange Rate Uncertainty

- Percentage changes in GBPUSD range from about -12% to +14.5%
- Mean monthly percentage change was -0.05%
- Standard deviation was 3.03%
- Would appear to be relatively normal

Assessing Exchange Rate Uncertainty

Daily Changes in USDZAR

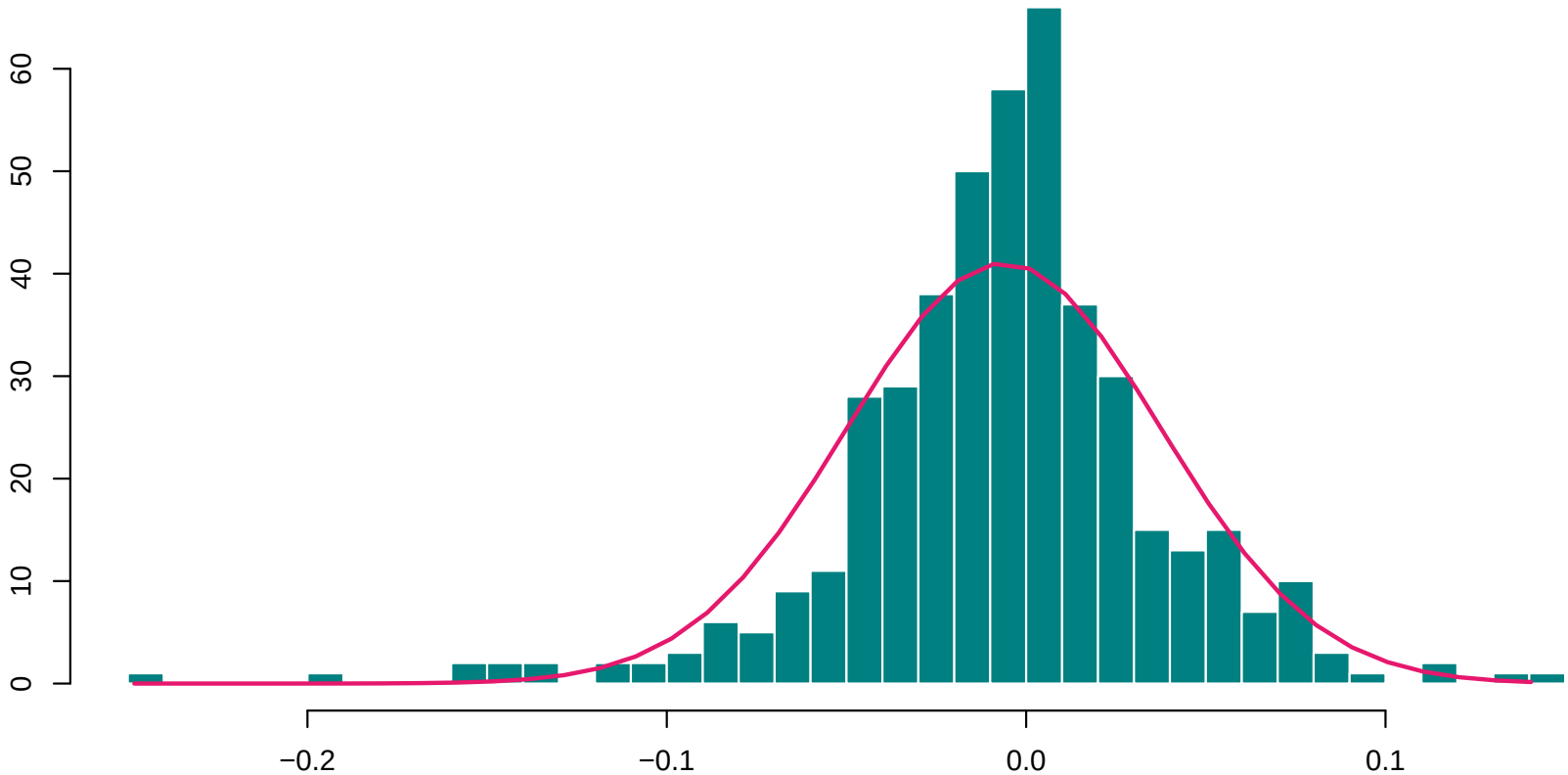

Figure: Source: Bloomberg

Assessing Exchange Rate Uncertainty

- Percentage changes in USDZAR have a large left-hand tail
- Large depreciations of the rand relative to the dollar are more frequent than large appreciations
- Histogram has relatively fat tails
- Would appear to be non-normal
- Similar to the currencies of other emerging market economies

Assessing Exchange Rate Uncertainty

- Financial managers are interested in the probability distribution of future spot exchange rates
- As the probability distribution of the future exchange rate depends on all the information available at time t, we would be interested in the conditional probability distribution
- This gives rise to the conditional mean and dispersion of the future exchange rate

Assessing Exchange Rate Uncertainty

- In the case of the GBPUSD rate we could make use of a normal distribution to calculate:

$$\begin{aligned} \text{Conditional mean} &= \left[S_t(1+\mu)\right] \\ \text{Conditional standard deviation} &= \left[S_t\sigma\right] \end{aligned}$$

- expected mean value of the change in the exchange rate is μ
- expected standard deviation of the change in the exchange rate is σ
- Can also determine the probability that the exchange rate in the future will be greater or less than a particular future spot rate

Assessing Exchange Rate Uncertainty

- For example:
- If the current GBPUSD exchange rate is \$1.50, and we expect that the pound will appreciate relative to the dollar by 2% over the next 90 days
- Conditional expectation of the future spot rate in 90 days is then

$$\left[1.50 \times (1 + 0.02)\right] = \$1.53$$

- If the standard deviation of the rate of appreciation over the next 90 days is expected to be 4%
- The standard deviation of the conditional distribution of the expected future spot exchange rate is

$$\left[1.50 \times 0.04\right] = \$0.06$$

Assessing Exchange Rate Uncertainty

- With these statistics we can determine the probability that the future exchange rate will fall within any given range
- For a normally distributed exchange rate, slightly more than two-thirds of the probability distribution is within plus or minus 1 standard deviation of the mean
- In our example, this range is from $\left[\ 1.53 \pm 0.06\ \right]$, which spans \$1.47 to \$1.59
- Given a probability distribution of future exchange rates, we can also determine the probability that the exchange rate in the future will be greater or less than a particular future spot rate
- For example, if we want to know how likely it is that the pound will strengthen over the next 90 days to at least \$1.60
  - Since \$1.60 is greater than the conditional mean of \$1.53, by \$0.07, and the standard deviation is \$0.06
  - We want to know how likely it is that we will be $[0.07/0.06] = 1.167$ standard deviations above the mean
- For the normal distribution, this probability is 12.16%

Assessing Exchange Rate Uncertainty

Exchange Rate	Standard Deviation	Forward Market Return
EURUSD	11.17	11.25
GBPUSD	10.57	10.7
USDJPY	11.66	11.81
EURJPY	11.34	11.42
EURGBP	9.25	9.35
GBPJPY	12.37	12.49

Table: Standard Deviation of Monthly Exchange Rate Changes and Forward Market Returns

Volatility Clustering

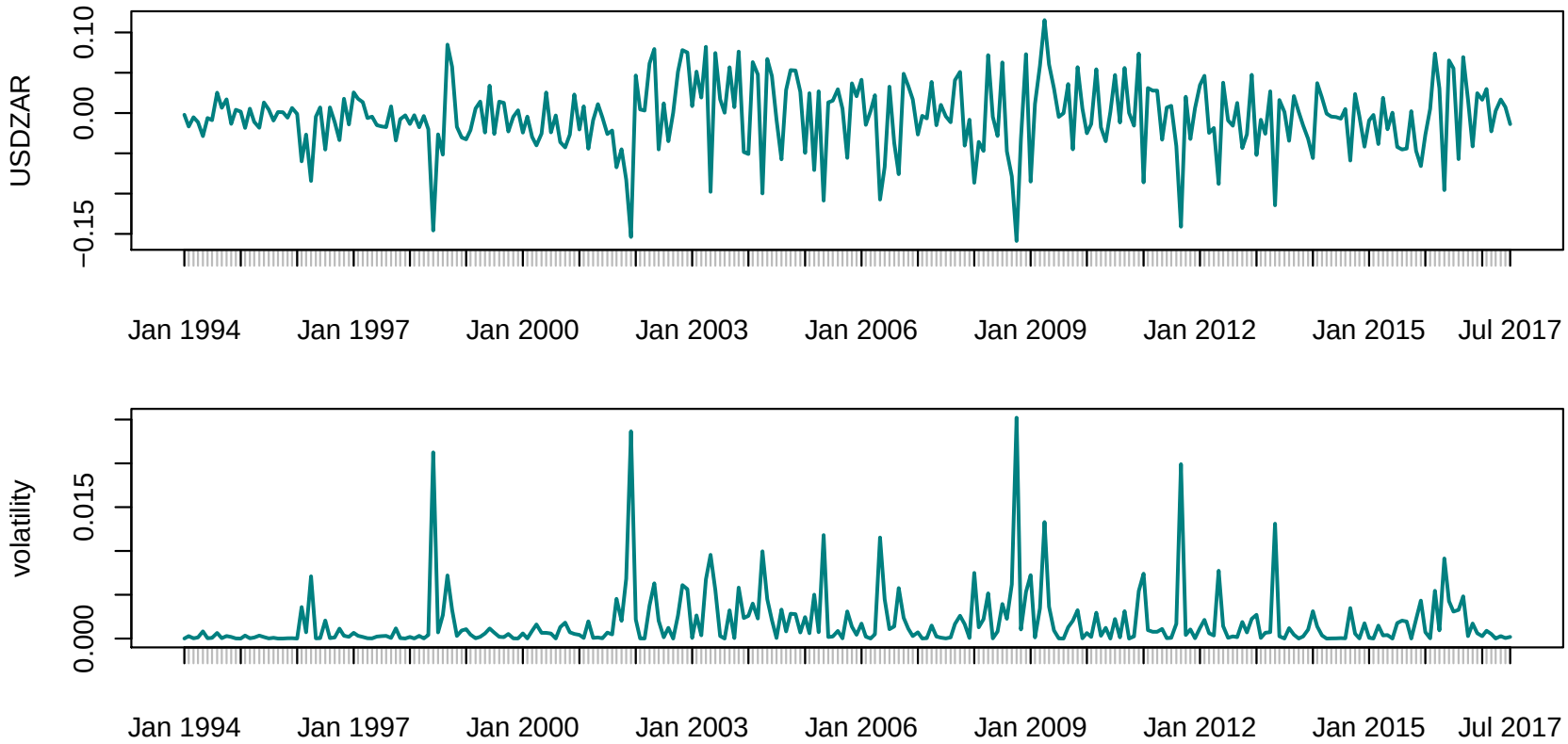

Figure: Monthly Changes and Volatility in USDZAR

Volatility Clustering

- Graph reveals relatively long quiet periods
- Also see volatility spikes in crises in Asia [1997], Russia [1998] and Argentina [1999], South African currency crisis [2001] and the global financial crisis [2008-2009]
- Could be modelled with a GARCH framework, where σ_t^2 is the conditional variance

$$\begin{aligned} y_t &= \mu + \sigma_t \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \sum_{k=1}^{K} \alpha_k (y_t - \mu)_{t-k}^2 + \sum_{j=1}^{J} \beta_j \sigma_{t-j}^2 \end{aligned}$$

- Model accommodates persistence in volatility
- Behaviour is consistent with most other financial assets

The Foreign Exchange Market

"Is it reasonable to expect exchange rate forecasts to be characterized with **large variability***? We think the answer is yes because exchange rates are the relative prices of currencies and currencies are assets. Thus, exchange rates are* **asset prices***, such as stock prices, and we should expect exchange rates to behave very much like other asset prices, such as stock prices, which are also very difficult to predict. If exchange rates were easy to predict,* **lots of easy money** *would be made betting that one currency would strengthen relative to another."*

Bekaert and Hodrick (2012)

Conclusion

- Making use of probability distributions allows us to quantify transaction exchange risk by considering the properties of historical data
- Not a perfect science as these measures of risk depend upon the estimate of volatility, which could vary over time