Financial Frictions

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Table of contents

Introduction

- The recent Great Recession highlighted the potential importance of financial market imperfections when modelling macroeconomic fluctuations
- In what follows we discusses how to incorporate financial market imperfections in a tractable way
- Modelling financial frictions in a fully micro-founded way is technically challenging
- If fully micro-founded, to get financial frictions into a model one needs some degree of market incompleteness and some level of heterogeneity

Introduction

- The financial system aggregates the savings of households and funnels it to finance investment in physical capital by firms
- This funnelling of savings can either be equity or debt, and if we do not include other frictions it does not matter which
- For standard DSGE models it doesn't matter if we think about the household making capital accumulation decisions rather than firms

Introduction

- Financial market imperfections, or financial frictions, generically refer to situations where the funnelling of saving from households to firms gets messed up somehow
- In the real world, a large fraction of firm financing comes form of conventional debt that is facilitated through financial intermediaries
- These financial intermediaries play important informational roles to make debt finance more efficient than equity finance
- For example, banks making loans know a lot more about a company than individuals who may purchase new equity

Introduction

- There are two popular ways to throw a spanner into the use of debt to finance operations
- The first dates back to Townsend (1979) and is known as the "cost state verification" (CSV) problem
- This has been brought to the fore in macroeconomic models in Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), and Bernanke, Gertler, and Gilchrist (1999)
- Here the problem involves information asymmetry, where owners have private information about their projects and limited liability

Introduction

- This gives rise to a contracting problem between a lender and firm
- Hence there is an external finance premium, wherein firms pay a higher interest rate to raise money from external sources
- Could also include conditions for collateral constraints, where the more net worth a firm has the more financing it can do internally
- Hence, fluctuations in asset prices (which affect net worth) affect firm investment decisions

Introduction

- The alternative setup is sometimes known as "costly enforcement" and dates back to Kiyotaki and Moore (1997)
- Here there is no asymmetric information *per se*, but there is costly enforcement
- In the event of default the lender only recovers a fraction of the debt
- Like the CSV framework, this also leads to a contracting problem
- Here we also have a collateral constraint wherein a borrower can only borrow a fraction of the value of his assets
- Hence, we can think about a financial market shock as a shock to the fraction of the value of assets a borrower can borrow
- Furthermore, asset price fluctuations will affect the borrowing limit
- In this way, just like in the CSV framework, fluctuations in asset prices can directly affect firm investment decisions

Introduction

- The collateral constraint in the model is much easier to deal with and provides a convenient way to think about financial frictions
- This is the route that we will follow, where we simply specify (without formally deriving) what a firm has to finance out of debt
- We then include a constraint in which total borrowing cannot exceed a fraction of the value of the firm's capital stock (or collateral)

Introduction

- This will allow us to illustrate most of the ideas of financial frictions
- We will think about a financial shock as an exogenous change in the fraction of the value of a firm's capital that it can borrow
- There are some downsides to the collateral constraint approach
  - There is no default in equilibrium (unlike in the CSV framework)
  - There are no credit spreads in the collateral constraint framework
  - Finally, we can't think about the role of idiosyncratic risk in the way one might with the CSV framework

Introduction

- Assume that the firm has to finance its payments to labour with an intraperiod loan (what is called working capital)
- Hence, the amount it can finance is tied to the value of its capital
- Because this financial friction directly affects the firm's choice of labour, it generates a labour wedge and financial shocks can induce co-movement among aggregate variables
- Thereafter, we consider the case where investment in new physical capital must be financed via either intra or interperiod debt
- In addition to a financial shock, we also analyse how the presence of a financial constraint impacts the responses to a productivity shock

Simple Collateral Constraint Model

Household

- The household problem is fairly standard
- We assume that the firm owns the capital stock and makes capital accumulation decisions
- We require that the firm borrow to finance its payments to labour (what is known as "working capital") via an intra-period loan
- The amount that the firm can borrow depends on the value of its capital stock

Simple Collateral Constraint Model

Household

- For this to make sense, we need the firm to own the capital stock
- The household problem is:

$$\max_{C_t, N_t, B_{t+1}} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \theta \frac{N_t^{1+\gamma}}{1+\gamma} \right]$$

- with budget constraint

$$C_t + B_{t+1} - B_t \leq w_t N_t + r_{t-1} B_t + \Pi_t$$

- The terms here are all standard
- B_t is the stock of savings/debt with which a household enters a period, with r_{t-1} the real interest rate on that savings
- Π_t is a dividend distribution from the firm

Simple Collateral Constraint Model

- After setting up the Lagrangian we can solve for the first-order conditions:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_t} &= \frac{1}{C_t} - \lambda_t = 0 \\
\frac{\partial \mathcal{L}}{\partial N_t} &= -\theta N_t^{\gamma} + \lambda_t w_t = 0 \\
\frac{\partial \mathcal{L}}{\partial B_{t+1}} &= -\lambda_t + \beta \mathbb{E}_t \lambda_{t+1} \left(1 + r_t\right) = 0
\end{aligned}
$$

Simple Collateral Constraint Model

Household

- These are standard and the multiplier can be eliminated, leaving:

$$
\begin{aligned}
\frac{1}{C_t} w_t &= \theta N_t^{\gamma} \\
\frac{\mathbb{E}_t C_{t+1}}{C_t} &= \beta \left(1 + r_t\right)
\end{aligned}
$$

Simple Collateral Constraint Model

- The firm produces output using a standard Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

- There is a capital adjustment cost, where capital accumulates according to:

$$K_{t+1} = I_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t + (1 - \delta) K_t$$

Simple Collateral Constraint Model

Firm

- Investment is financed out of dividends, and for simplicity we assume that the firm issues no intertemporal debt
- The dividend payout is then:

$$\Pi_t = A_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - I_t$$

Simple Collateral Constraint Model

Financial Friction

- What differentiates the setup relative to a frictionless model is the inclusion of a working capital constraint
- We assume that the firm must pay labour in advance of producing output, and finances this labour payment with an intraperiod loan
- Since the loan is intratemporal (as opposed to the standard intertemporal debt), there is no interest on this loan

Simple Collateral Constraint Model

Financial Friction

- We assume that the amount the firm can borrow equals a fraction of the value of its capital
- In particular, the firm faces the following constraint:

$$w_t N_t \leq \xi_t q_t K_t$$

- Here, $w_t N_t$ denotes total wage payments, q_t is the value of capital, K_t is how much capital the firm has, and ξ_t is an exogenous and stochastic borrowing limit
- The basic idea is that if the borrower defaults on the working capital loan, the lender can only seize a fraction, $\xi_t < 1$, of the firm's assets
- This setup is sometimes known as the costly enforcement model
- Since a firm might renege on its debt, a lender will only allow a firm to borrow up to a fraction of its debt

Simple Collateral Constraint Model

Financial Friction

- The Lagrangian for the firm allows for it to discount future profit flows by the stochastic discount factor of the household, $\mathbb{E}_t \beta^j \frac{\lambda_{t+j}}{\lambda_t}$
- Let q_t be the multiplier on the accumulation equation
- Since the units of the firm's problem are goods, q_t has the interpretation of reflecting how many goods the firm would give up for an additional unit of installed capital

Simple Collateral Constraint Model

- The firm's Lagrangian is:

$$\mathcal{L}_{N_t,I_t,K_{t+1}} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ \left[A_{t+j} K_{t+j}^{\alpha} N_{t+j}^{1-\alpha} - w_{t+j} N_{t+j} - I_{t+j} \right] \ldots \right.$$

$$\cdots + q_t \left[I_{t+j} - \frac{\phi}{2} \left(\frac{I_{t+j}}{K_{t+j}} - \delta \right)^2 K_{t+j} + (1-\delta) K_{t+j} - K_{t+j+1} \right]$$

$$\left. \ldots + \mu_{t+j} \left[\xi_{t+j} q_{t+j} K_{t+j} - w_{t+j} N_{t+j} \right] \right\}$$

Simple Collateral Constraint Model

- The first order conditions are:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial N_t} &= (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} - w_t(1+\mu_t) = 0 \\
\frac{\partial \mathcal{L}}{\partial I_t} &= 1 - q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta \right) \right] = 0 \\
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= -q_t + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \Big[\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + \ldots \\
& \qquad \ldots q_t \left((1-\delta) + \mu_{t+1}\xi_{t+1} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \ldots \right. \\
& \qquad \left. \ldots + \phi \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) \Big] = 0
\end{aligned}
$$

Simple Collateral Constraint Model

- Note that if the collateral constraint did not bind (i.e. $\mu_t = 0$), then we would have identical conditions to a standard model with a capital adjustment cost
- One can see here that if the constraint does bind, however, then there is a wedge between the wage and the marginal product of labour
- If the constraint becomes "tighter", we would expect the multiplier on the constraint to become bigger, which will function much like an increase in a tax on labour income

Simple Collateral Constraint Model

- The market-clearing conditions are standard
- Since the firm issues no intertemporal debt, then the household cannot have any stock of savings
- The full set of equilibrium conditions can therefore be written:

$$
\begin{aligned}
\frac{1}{C_t} &= \lambda_t \\
\theta N_t^{\gamma} &= \lambda_t w_t \\
\lambda_t &= \beta \mathbb{E}_t \lambda_{t+1} \left(1 + r_t\right) \\
(1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} &= w_t (1 + \mu_t) \\
1 &= q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta \right) \right] \\
q_t &= \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \Big[\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + \ldots \\
& \quad \ldots q_t \left((1-\delta) + \mu_{t+1} \xi_{t+1} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \ldots \right. \\
& \quad \left. \ldots + \phi \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) \Big]
\end{aligned}
$$

Simple Collateral Constraint Model

$$
\begin{aligned}
Y_t &= A_t K_t^{\alpha} N_t^{1-\alpha} \\
Y_t &= C_t + I_t + G_t \\
K_{t+1} &= I_t - \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + (1-\delta)K_t \\
w_t N_t &\leq \xi_t q_t K_t \\
\log A_t &= (1-\rho_A)\log A + \rho_A \log A_{t-1} + s_A \varepsilon_{A,t} \\
\log \xi_t &= (1-\rho_\xi)\log \xi + \rho_\xi \log \xi_{t-1} + s_\xi \varepsilon_{\xi,t}
\end{aligned}
$$

Note that there are twelve equations and twelve variables:
$Ct, \lambda_t, w_t, N_t, \mu_t, r_t, K_t, A_t, Y_t, I_t, \xi_t,$ and q_t

Simple Collateral Constraint Model

- Let us solve for the non-stochastic steady state
- Variables without time subscripts denote steady state values
- We will consider two regions - one in which the collateral constraints binds and the other in which it does not
- If the constraint does not bind, then $\mu = 0$ and the steady state is the same as in the standard model

Simple Collateral Constraint Model

- From the first order condition for investment as well as the capital accumulation equation, it is clear that $q = 1$
- We can then solve for the steady state capital-labour ratio from the dynamic capital supply Euler equation:

$$\frac{K}{N} = \left(\frac{\alpha}{\frac{1}{\beta} - (1-\delta) - \mu\xi} \right)^{\frac{1}{1-\alpha}}$$

Simple Collateral Constraint Model

Steady State

- Note that this expression holds regardless of whether the constraint binds or not
- If the constraint does not bind, then this is the usual expression for the capital-labour ratio
- If the constraint binds, then $\mu > 0$ and the capital-labour ratio will be larger than in the standard case
- This may seem odd that capital-labour would be larger in the constrained case, but note that the constraint applies to labour
- So a more binding constraint will tend to reduce N, which drives up the capital-labour ratio

Simple Collateral Constraint Model

- How would we know what μ is?
- It is either 0 (non-binding) or positive
- Suppose that the collateral constraint binds
- Then from the constraint evaluated in steady state we know:

$$w = \xi \left(\frac{K}{N} \right)$$

- From the first order condition for labour demand, we also know that:

$$w = \frac{1}{1+\mu}(1-\alpha)\left(\frac{K}{N} \right)^{\alpha}$$

- Combining these, we have:

$$\xi\left(\frac{K}{N}\right)=\frac{1}{1+\mu}(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

- Or

$$\xi\left(1+\mu\right)=\left(1-\alpha\right)\left(\frac{K}{N}\right)^{\alpha-1}$$

- Using what we now know about $\frac{K}{N}$ this can be written:

$$\xi\left(1+\mu\right)=\left(1-\alpha\right)\left(\frac{\frac{1}{\beta}-\left(1-\delta\right)-\mu\xi}{\alpha}\right)$$

Simple Collateral Constraint Model

Steady State

- Then solving for μ, we get:

$$\mu = \frac{1-\alpha}{\xi}\left(\frac{1}{\beta} - (1-\delta)\right) - \alpha$$

- We can see here that the multiplier is decreasing in ξ, which is the fraction of the value of capital that can be borrowed against
- This makes sense - the bigger is the fraction, the "less binding" is the collateral constraint, and hence the lower will be the multiplier
- We can also solve for ξ when the constraint is binding

Simple Collateral Constraint Model

Steady State

- This is is found by solving for the values of ξ when $\mu > 0$:

$$\xi < \frac{1-\alpha}{\alpha}\left(\frac{1}{\beta} - (1-\delta)\right)$$

- If ξ is bigger than this, then the constraint will not bind and we will be back in the normal case

Simple Collateral Constraint Model

Steady State

- Now let us proceed by combining the labour supply and labour demand conditions evaluated in the steady state:

$$\theta N^{\gamma} = \frac{1}{C}\frac{1}{1+\mu}(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

- Multiply both sides by N:

$$\theta N^{1+\gamma} = \frac{N}{C}\frac{1}{1+\mu}(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

Simple Collateral Constraint Model

Steady State

- From the resource constraint, we know that:

$$\frac{C}{N} = \left(\frac{K}{N}\right)^{\alpha} - \delta \frac{K}{N}$$

- Hence, we can solve for N as:

$$N = \left(\frac{1}{\theta} \frac{1}{1+\mu} \frac{(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}}{\left(\frac{K}{N}\right)^{\alpha} - \delta \frac{K}{N}} \right)^{\frac{1}{1+\gamma}}$$

Simple Collateral Constraint Model

- How does the value of ξ affect N?
- There is both a direct effect and an indirect effect
- A higher ξ makes μ smaller, which makes $(1+\mu)^{-1}$ bigger, and therefore works to make N bigger
- We would therefore expect N to be increasing in ξ
- Once we know N, it is straightforward to solve for the rest of the steady state

Simple Collateral Constraint Model

Impulse Responses

- Assume the following parameter values: $\beta = 0.99$, $\alpha = 1/3$, $\delta = 0.02$, $\phi = 4$, $\theta = 7.71$, $\gamma = 1$, $\rho_A = 0.98$, $s_A = 0.01$, $\rho_\xi = 0.90$, and $s_\xi = 0.01$
- If the collateral constraint were not binding, with these parameter values labour hours would be $1/3$ in steady state
- With these parameter values, the cutoff value of ξ where the constraint binds is $\xi = 0.0602$
- Because we want to solve the model using perturbation techniques, we want to approximate about a point where the constraint binds, and then implicitly ignore any possibility of the constraint perhaps not binding at some point in the future
- Therefore, assume $\xi = 0.05$, which means that in the steady state the constraint binds

Simple Collateral Constraint Model

Impulse Responses

- In what follows, we show impulse responses for two cases:
    - one where the constraint binds (for the parameterization above)
    - another in which it does not
- This allows us to compare how the economy reacts to the two kinds of shocks with and without the constraint binding

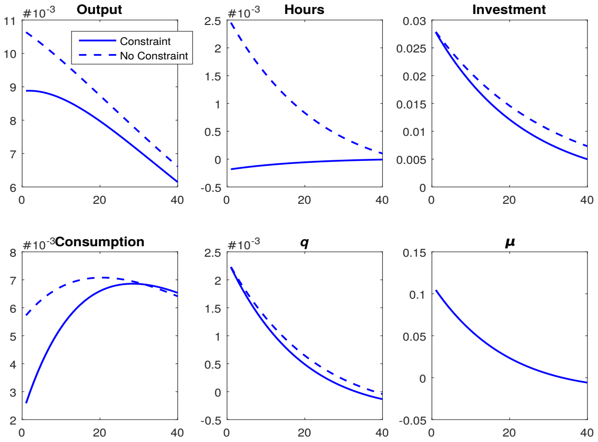

Figure: 1 - Productivity Shock

Simple Collateral Constraint Model

- First, note that μ_t increases significantly after the productivity shock
- This means that the collateral constraint is "tighter"
- Increase in q_t is pretty similar whether the constraint binds or not
- Hence, what the increase in μ_t means is that the firm would like to increase total payments to labour $(w_t N_t)$ by more than the value of the firm's capital increases
- The increase in μ_t is isomorphic to an increase in the tax on labour
- Therefore N_t increases less than it would in the unconstrained case
- Here N_t decreases on impact in the constrained case
- And output goes up by significantly less than the unconstrained case

Simple Collateral Constraint Model

- Even though output goes up by less in the constrained case, the impact response of investment is fairly similar
- What accounts for this? The firm has an incentive to increase its investment by a lot, because this results in more capital in the future, which is subject to a constraint
- So investment today helps relax the credit constraint in the future
- Hence, investment increases similarly after the productivity shock regardless of whether the collateral constraint binds or not
- Since output goes up by significantly less, but the investment response is roughly the same, it follows that consumption increases by significantly less when the collateral constraint binds

Simple Collateral Constraint Model

- Hence, the collateral constraint here dampens the responses to a productivity shock
- Is there a so-called "financial accelerator" mechanism present in the model?
- The basic idea of the financial accelerator is that exogenous shocks (such as productivity shocks) impact the value of capital, q_t, which affects the ability to borrow (i.e. credit markets)
- This is somewhat mechanical here - changes in q_t affect the borrowing limit
- Hence, the idea of the financial accelerator is that exogenous shocks might have bigger effects on output through an effect on asset prices and hence credit conditions than if financial frictions were absent
- Given that output responds significantly less to the productivity shock when the constraint binds than when it does not, that does not seem to be the case here

Simple Collateral Constraint Model

Impulse Responses

- To better see if there is a financial accelerator mechanism, it is useful to consider two cases where the financial constraint binds in both
- But in one, assume that $\phi = 0$, so that $q_t = 1$
- In the other, let $\phi > 0$
- The responses are shown below

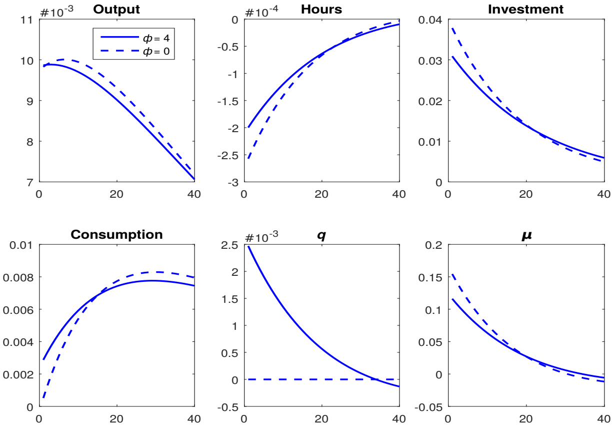

Figure: 2 - Productivity Shock (Adjustment Cost vs. No Adjustment Cost)

Simple Collateral Constraint Model

Impulse Responses

- The difference between these two cases, of course, is that q_t can react when $\phi > 0$, and it is constant when there is no adjustment cost
- If you look closely enough at the figure, you will see that output reacts more on impact (but only by a little) when $\phi = 4$ than when $\phi = 0$
- What is happening here is that N_t falls by less

Simple Collateral Constraint Model

- When $\phi = 0$, total labour payments cannot change on impact because $q_t K_t$ is then predetermined
- But when $\phi > 0$, $w_t N_t$ can go up on impact, even though K_t cannot
- This means that if the shock can affect q_t (the price of capital), it can have a bigger effect than if it cannot
- This is loosely a financial accelerator mechanism
- The financial accelerator mechanism here turns out to be pretty weak, which is often true in models of this sort
- What drives this is that the model does not predict a very big increase in q_t after the productivity shock
- If the model was able to generate a huge amount of asset price volatility, the financial accelerator mechanism would be less weak

Simple Collateral Constraint Model

Impulse Responses

- Another aspect worth observing when looking at the figures together
- Note that the model with a binding collateral constraint produces a bit of a hump-shaped response of output to the productivity shock
- Indeed, in Figure 2 we see that the apparent hump-shape is even stronger without an adjustment cost to capital
- What is driving this?

Simple Collateral Constraint Model

- For this, it is easiest to focus on the responses shown in Figure 1
- When the constraint binds, the output response is much smaller than when the constraint does not bind
- The same is not true for investment
- Put differently, the binding collateral constraint results in an increase in the volatility of investment relative to output
- Hence, when productivity improves the firm would like to hire more labour (regardless of the constraint)
- But it would also like to ease the constraint in the future by accumulating more capital in the present

Simple Collateral Constraint Model

- While this won't let the firm hire more labour immediately, it will once the capital comes on line
- This means that the firm has an incentive to do more investment, which strengthens the model's internal propagation mechanism
- The extra capital that is accumulated (relative to the initial increase in output) generates more interesting output dynamics than in the model with no collateral constraint
- Put another way, binding financial constraints often have the effect of increasing propagation and persistence, in a way somewhat similar to the inclusion of capital/investment adjustment costs
- This is an older point that is made in Carlstrom & Fuerst (1997) in the context of a costly state verification (CSV) setup

Simple Collateral Constraint Model

- Next, we consider the responses to a financial shock (a shock to ξ_t)
- This is obviously only relevant in the situation in which the collateral constraint binds
- The responses are shown below in Figure 3

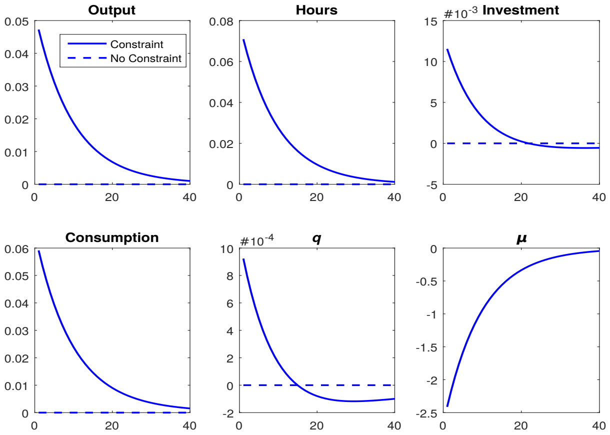

Figure: 3 - Financial Shock

Simple Collateral Constraint Model

- The increase in ξ_t makes the collateral constraint "less binding" and consequently results in a fall in μ_t
- Mechanically in terms of how the other equations of the model look, this looks like a decrease in a distortionary tax rate on labour income
- As a result, firms hire more labour and output rises, which results in higher consumption
- Investment (and also q_t) also rise, albeit not by very much
- Hence, a shock to the fraction of collateral a firm can borrow against for working capital (what we can loosely interpret as a financial shock) can lead to a broad-based movement in economic activity
- It can do so precisely because it generates a labour wedge so that it presents results that are similar a time-varying tax on labour income

Alternative Setups

- We can now consider two alternative setups
- These involve different factors that must be financed by debt, and hence different aspects will be affected by the collateral constraint
- First, we assume that both payments to labour and investment must be financed by working capital
- Second, we assume there is no working capital requirement, but that investment must be financed with intertemporal debt
- Thus intertemporal debt is subject to the collateral requirement

Investment Financed by Working Capital as Well

- Let's consider the same setup, but now we assume the firm uses working capital to finance its investment, as well as its labour input
- This leaves the firm problem unaffected relative to our baseline setup, but the collateral constraint is now:

$$w_t N_t + I_t \leq \xi_t q_t K_t$$

Investment Financed by Working Capital as Well

- The only first order condition affected by this difference is the one over investment, which now becomes:

$$1 + \mu_t = q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta \right) \right]$$

- This conveys something interesting
- In particular, even if $\phi = 0$, the price of capital, q_t, could be affected by the collateral constraint when it binds

Investment Financed by Working Capital as Well

Steady State

- Let's solve for the steady state of the revised problem
- From the accumulation equation we still have $I = \delta K$
- This means that steady state q is:

$$q = 1 + \mu$$

- The expression for the steady state capital stock is the same

$$\frac{K}{N} = \left(\frac{\alpha}{\frac{1}{\beta} - (1 - \delta) - \mu\xi} \right)^{\frac{1}{1-\alpha}}$$

Investment Financed by Working Capital as Well

- As before, begin by assuming that the collateral constraint binds
- With $I = \delta K$, after plugging in for q we can then get an expression for the real wage steady state:

$$w = \left(\xi\left(1+\mu\right) - \delta\right)\left(\frac{K}{N}\right)$$

- From the labour demand condition, we also know that:

$$w = \frac{1}{1+\mu}(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}$$

Investment Financed by Working Capital as Well

Steady State

- Then we have:

$$\frac{1}{1+\mu}(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}=\left(\xi\left(1+\mu\right)-\delta\right)\left(\frac{K}{N}\right)$$

- This can be written:

$$\frac{1}{1+\mu}(1-\alpha)=\left(\xi\left(1+\mu\right)-\delta\right)\left(\frac{K}{N}\right)^{1-\alpha}$$

- Given that $\frac{K}{N}$ is now known as a function of μ, this expression can be solved for μ

Investment Financed by Working Capital as Well

Impulse Responses

- We can then solve the model using the same parameterization for where the collateral constraint only applies to working capital for labour payments
- These impulse responses for a productivity shock are shown in Figure 4

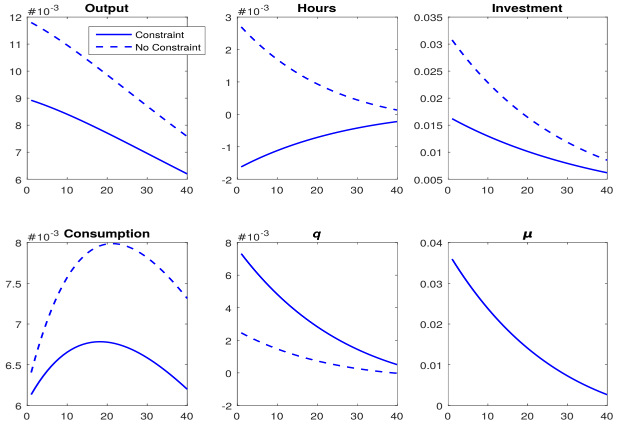

Figure: 4 - Productivity Shock

Investment Financed by Working Capital as Well

Impulse Responses

- When productivity increases, the firm would like to hire more labour and do more investment, so the constraint tightens
- Consequently, μ_t increases
- Similarly to the previous case, hours worked declines on impact
- Differently than before, investment increases substantially less (relative to the unconstrained case)
- This is kind of mechanical given that investment now must be financed via working capital
- The smaller increase in investment means that we get less propagation of the productivity shock relative to the case where working capital is only needed for labour payments
- Hence, the output impulse response is not as hump-shaped

Investment Financed by Working Capital as Well

Impulse Responses

- Next, consider a financial shock
- This eases the constraint, and hence results in μ_t falling
- Differently than above, this results in q_t falling, not rising
- Essentially, capital is less valuable
- In spite of the decline in q_t, investment rises (because of the decline in μ_t)
- Consumption also declines, rather than rises, on impact here
- The way to think about this is that prior to the financial shock, the economy was investing less (and hence consuming more) than it would like to in an unconstrained world
- The easing of the constraint (when it applies to both payments to labour and investment) induces a substitution away from private consumption towards investment

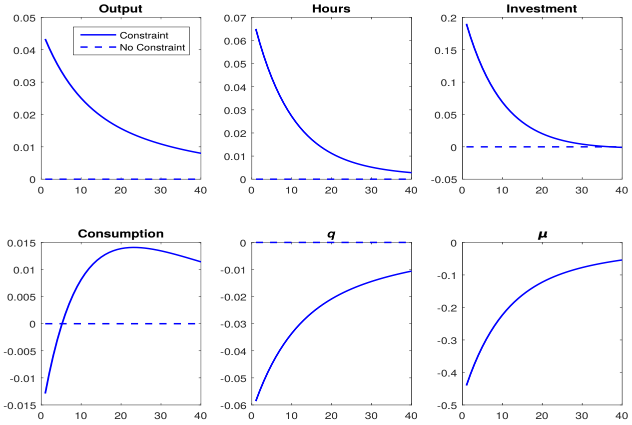

Figure: 5 - Financial Shock

Collateral Constraint Applies to Intertemporal Debt

- Suppose that, for unmodeled reasons, the firm finances all investment with new one period intertemporal debt (as opposed to the intratemporal debt considered in the working capital cases)
- That is, suppose that each period the firm issues new debt, D_{t+1}, to finance its current investment
- In other words, we must have:

$$I_t = D_{t+1}$$

Collateral Constraint Applies to Intertemporal Debt

- Each period, the firm is required to pay off principle plus interest on the debt it brought into the period
- This amounts to paying off $(1 + r_{t-1})I_{t-1}$
- Hence, the dividend the firm pays out is:

$$\Pi_t = Y_t - w_t N_t - (1 + r_{t-1})I_t$$

Collateral Constraint Applies to Intertemporal Debt

- Now, the collateral constraint applies only to intertemporal debt
- In particular:

$$D_{t+1} \leq \xi_t q_t K_t$$

- Since $D_{t+1} = I_t$, this means that the firm's current investment is constrained by the value of its existing capital
- The accumulation equation is the same as before

Collateral Constraint Applies to Intertemporal Debt

- The Lagrangian is now:

$$\mathcal{L} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} \left\{ \left[A_{t+j} K_{t+j}^{\alpha} N_{t+j}^{1-\alpha} - w_{t+j} N_{t+j} - (1 + r_{t-1}) I_{t+j} \right] \ldots \right.$$

$$\cdots + q_t \left[I_{t+j} - \frac{\phi}{2} \left(\frac{I_{t+j}}{K_{t+j}} - \delta \right)^2 K_{t+j} + (1 - \delta) K_{t+j} - K_{t+j+1} \right]$$

$$\left. + \mu_{t+j} \left[\xi_{t+j} q_{t+j} K_{t+j} - I_{t+j} \right] \right\}$$

Collateral Constraint Applies to Intertemporal Debt

- The first order conditions are:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial N_t} &= (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} - w_t = 0 \\
\frac{\partial \mathcal{L}}{\partial I_t} &= \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} (1 + r_t) - q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta \right) \right] = 0 \\
\frac{\partial \mathcal{L}}{\partial K_{t+1}} &= -q_t + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \Big[\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + \ldots \\
& \ldots q_t \left((1-\delta) + \mu_{t+1} \xi_{t+1} - \frac{\phi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right)^2 \ldots \right. \\
& \left. \ldots + \phi \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \frac{I_{t+1}}{K_{t+1}} \right) \Big] = 0
\end{aligned}
$$

Collateral Constraint Applies to Intertemporal Debt

- The first order condition with respect to K_t is the same as before
- The first order condition for N_t now looks standard, without any wedge between the wage and the marginal product of labour
- What is slightly different than before is the first order condition with respect to investment
- From the household's first order condition, we know that $\beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}(1+r_t) = 1$
- Hence this condition can be written:

$$1 + \mu_t = q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta \right) \right]$$

Collateral Constraint Applies to Intertemporal Debt

- This looks identical to the case where investment must be financed via working capital (as opposed to an intertemporal loan)
- If the collateral constraint does not bind, then $\mu_t = 0$, and this is the same first order condition as before (or as what would emerge in the standard model)
- This is simply Modigliani-Miller - if there is no financial constraint, the equilibrium conditions will be identical if we force the firm to finance investment with debt (this case) or equity (the earlier case considered when there was a working capital constraint on labour)

Collateral Constraint Applies to Intertemporal Debt

- The household holds all debt issued by the firm
- The full set of equilibrium conditions can therefore be written:

$$
\begin{aligned}
\frac{1}{C_t} &= \lambda_t \\
\theta N_t^{\gamma} &= \lambda_t w_t \\
\lambda_t &= \beta \mathbb{E}_t \lambda_{t+1} \left(1 + r_t\right) \\
(1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} &= w_t \\
(1 + \mu_t) &= q_t \left[1 - \phi \left(\frac{I_t}{K_t} - \delta\right)\right] \\
q_t &= \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \Big[\alpha A_{t+1} K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} + \ldots \\
&\quad \ldots q_t \left((1-\delta) + \mu_{t+1}\xi_{t+1} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)^2 \ldots \right. \\
&\quad \left. \ldots + \phi \left(\frac{I_{t+1}}{K_{t+1}} - \delta\right) \frac{I_{t+1}}{K_{t+1}}\right)\Big]
\end{aligned}
$$

Collateral Constraint Applies to Intertemporal Debt

$$
\begin{aligned}
Y_t &= A_t K_t^{\alpha} N_t^{1-\alpha} \\
Y_t &= C_t + I_t + G_t \\
K_{t+1} &= I_t - \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + (1-\delta)K_t \\
I_t &\leq \xi_t q_t K_t \\
\log A_t &= (1-\rho_A)\log A + \rho_A \log A_{t-1} + s_A \varepsilon_{A,t} \\
\log \xi_t &= (1-\rho_\xi)\log \xi + \rho_\xi \log \xi_{t-1} + s_\xi \varepsilon_{\xi,t}
\end{aligned}
$$

Collateral Constraint Applies to Intertemporal Debt

- What we see here is that there really isn't a difference between requiring the firm to finance its investment with an intratemporal loan or intertemporal debt
- For investment, the first order conditions are the same

Collateral Constraint Applies to Intertemporal Debt

Steady State

- We need to solve for the non-stochastic steady state
- The capital-labour ratio in steady state is the same as in the working capital constraint model:

$$\frac{K}{N} = \left(\frac{\alpha}{\frac{1}{\beta} - (1-\delta) - \mu\xi} \right)^{\frac{1}{1-\alpha}}$$

- The wage can be determined directly from this
- From the accumulation equation, we also know that $I = \delta K$

Collateral Constraint Applies to Intertemporal Debt

- From the first order condition for investment, we then have:

$$1 + \mu = q$$

- In other words, if the constraint binds, $q > 1$
- Further, q_t could fluctuate outside of steady state even without an adjustment cost
- In a sense, similar to the discussion above, the collateral constraint on intertemporal debt functions like an adjustment cost

Collateral Constraint Applies to Intertemporal Debt

Steady State

- From the collateral constraint at equality, using $I = \delta K$, we get:

$$\delta = \xi(1 + \mu)$$

- This means that:

$$\mu = \frac{\delta}{\xi} - 1$$

Collateral Constraint Applies to Intertemporal Debt

Steady State

- Hence, for the collateral constraint to bind, we must have $\xi < \delta$ (so that $\mu > 0$)
- Once we have μ, we can then have $\frac{K}{N}$, and we can use that to solve for N as:

$$N = \left(\frac{1}{\theta} \frac{(1-\alpha)\left(\frac{K}{N}\right)^{\alpha}}{\left(\frac{K}{N}\right)^{\alpha} - \delta \frac{K}{N}} \right)^{\frac{1}{1+\gamma}}$$

- This is similar to the case where the collateral constraint applies to working capital, but there is no explicit μ term present

Collateral Constraint Applies to Intertemporal Debt

Impulse Responses

- The model is parameterized as above where we set $\xi = 0.015$, which ensures that the collateral constraint binds in the steady state
- Impulse responses to the productivity shock (Figure 6) and financial shock (Figure 7) are shown below:

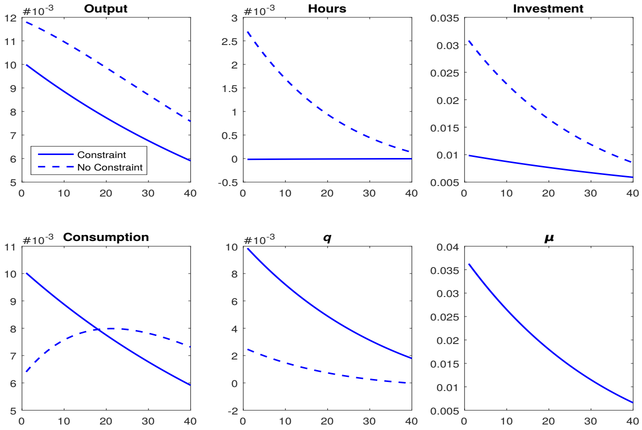

Figure: 6 - Productivity Shock

Collateral Constraint Applies to Intertemporal Debt

Impulse Responses

- We can see here that the intertemporal collateral constraint dampens the output response to a productivity shock
- When productivity increases, μ_t increases (essentially, the collateral constraint becomes tighter)
- The firm would like to do more investment, but can only increase its investment by the amount that its capital increases in value
- Labour hours essentially do not react at any horizon, meaning that the output response is close to equal to the increase in exogenous productivity
- In contrast, consumption increases by more in the economy with the constraint than without

Collateral Constraint Applies to Intertemporal Debt

- What is going on?
- When productivity improves, the firm would like to increase its investment, but is unable to do so by much because of the constraint binding
- Since investment can't go up by much, consumption goes up by more
- But consumption going up by more means that hours go up by less (there is a bigger offsetting wealth effect on labour supply), leaving labour hours roughly unchanged

Collateral Constraint Applies to Intertemporal Debt

- Next, consider the responses to the financial shock
- When ξ_t increases, μ_t falls (the constraint becomes looser)
- This results in a decline in q_t - capital is less valuable because it's not as necessary to ease the collateral constraint
- Investment increases, in a rather mechanical way - you can borrow more, so you invest more
- Note that investment and q_t no longer move in the same direction here, and so consumption declines
- The reason consumption declines is that you want to do more investment - essentially, prior to the shock the economy is constrained from doing as much investment as it wants, and the easing of the constraint means that you substitute away from consumption and into investment
- The fall in consumption results in hours going up (and the wage going down, though this is not shown)
- As a result of the increase in hours, output goes up

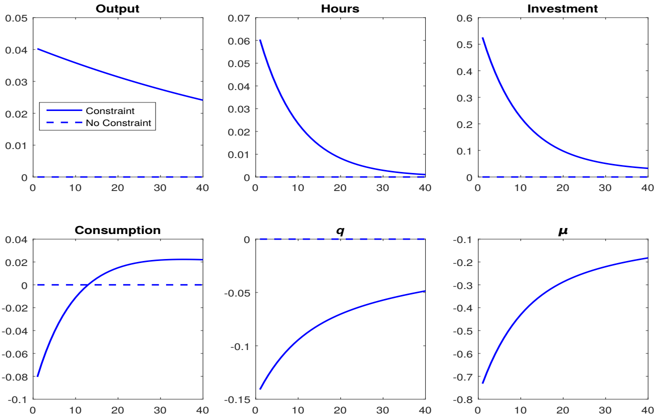

Figure: 7 - Financial Shock

Collateral Constraint Applies to Intertemporal Debt

Impulse Responses

- You will note that these responses look very similar to the case where both labour and investment must be financed via working capital

Conclusion

- One of the most straightforward ways of including financial frictions in the model is with the aid of conditions where the firm has to finance its payments to labour with an intraperiod loan (what is called working capital)
- In addition, we also constrain the firm's ability to obtain finance, which is tied to the value of capital stock
- we also consider cases where investment in new physical capital is financed via either intra or interperiod debt
- This will also lead to interesting dynamics for both a shock to the financial friction and to productivity